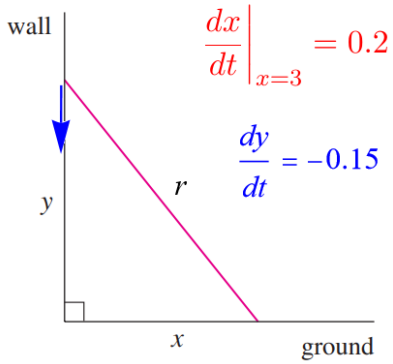
Exercise 33

The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

Solution

Draw a schematic of the ladder at a certain time.



Since we know dx/dt and dy/dt and want to know r, start with the Pythagorean theorem.

$$r^2 = x^2 + y^2$$

Take the derivative of both sides with respect to t by using the chain rule.

$$\frac{d}{dt}(r^2) = \frac{d}{dt}(x^2 + y^2)$$

$$2r \cdot \frac{dr}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

The ladder's length is a constant, so $dr/dt = 0$.

$$r(0) = (3)(0.2) + y(-0.15)$$

Solve for y.

$$0 = 0.6 - 0.15y$$

$$0.15y = 0.6$$

$$y = 4$$

Therefore, the length of the ladder is

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5 \text{ meters.}$$